Final Review
Sections Covered:

Ch 7: 7.1 – 7.7
Ch 8: 8.1, 8.2, 8.4
Ch 9: 9.1 – 9.4
Ch 10: 10.1 – 10.7
Ch 11: 11.1 – 11.4

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Chapter 7:

7.1: Simplify Rational Expressions (p. 440)

- Completely factor the numerator and denominator
- Divide out factors common to numerator and denominator
- Example: $\frac{x^2-16}{x^2+x-20} = \frac{(x+4)(x-4)}{(x+5)(x-4)} = \frac{x+4}{x+5}\cdot 1 = \frac{x+4}{x+5}$
- Try these on page 447: 3, 7, 9, 17, 23, 27, 31, 39
  (Solutions start on page 297 of the solution manual)

7.2: Multiplying and Dividing Rational Expressions (p. 451)

Multiplication:

- Completely factor numerator and denominator of both fractions
- Multiply the numerators and multiply denominators
  (You don't need a common denominator – only for addition & subtraction)
- Divide out common factors to simplify.
- Example: $\frac{a^2-4a+4}{a^2-4}\cdot\frac{a+3}{a-2} \Rightarrow \frac{(a-2)(a-2)}{(a+2)(a-2)}\cdot\frac{(a+3)}{(a-2)} \Rightarrow \frac{(a-2)(a-2)(a+3)}{(a+2)(a-2)(a-2)} \Rightarrow \frac{a+3}{a+2}$

Division:

- Multiply the first rational expression by the reciprocal of the second rational expression.
  - a.k.a. skip, flip, and multiply!
  - reciprocal means flipping the fraction (reciprocal of $\frac{a}{b} = \frac{b}{a}$)
- Example: $\frac{x^2-y^2}{3x^2+3xy} \div \frac{3x^2-2xy-y^2}{3x^2+6x} \Rightarrow \frac{(x+y)(x-y)}{3x(x+y)} \div \frac{(3x+y)(x-y)}{3x(x+2)} \Rightarrow \frac{(x+y)(x-y)}{3x(x+y)}\cdot\frac{3x(x+2)}{(3x+y)(x-y)}$

  Flip the second fraction

  Skip the first fraction

  $\Rightarrow \frac{(x+y)(x-y)(3x)(x+2)}{(3x)(x+y)(3x+y)(x-y)} \Rightarrow \frac{x+2}{3x+y}$
- Try these on page 458: 1, 7, 11, 15, 23, 25, 27, 57
  (Solutions start on page 301 of solutions manual)

7.3: Adding & Subtracting Rational Expressions (p. 460)

- If you already have a common denominator:
  - Add or subtract the numerators and place the sum or difference over the common denominator.
  - Example: $\frac{3x-1}{x^2+5x-6} - \frac{2x-7}{x^2+5x-6} \Rightarrow \frac{3x-1-(2x-7)}{x^2+5x-6} \Rightarrow \frac{3x-1-2x+7}{x^2+5x-6} \Rightarrow \frac{x+6}{x^2+5x-6} \Rightarrow \frac{x+6}{(x+6)(x-1)} \Rightarrow \frac{1}{x-1}$

- If you need to find the least common denominator (LCD)
  - Factor each denominator (including GCF if possible)
  - The LCD is the product of all unique factors found in the denominators raised to the power of the greatest number of times the factor appears in any one denominator
    ~ meaning if one denominator has the factor $(x+1)$ once and another has $(x+1)(x+1)$, then the LCD would include $(x+1)(x+1)$, or $(x+1)^2$ since the most times it appeared in one denominator is twice.
  - Example: Find the LCD of $\frac{9x^2}{7x-14}$, $\frac{6x}{14(x-2)^2}$

    $\frac{9x^2}{7x-14} \Rightarrow \frac{9x^2}{7(x-2)}$ Denominator factors: $7(x-2)$

    $\frac{6x}{14(x-2)^2} \Rightarrow \frac{6x}{7\cdot 2(x-2)(x-2)}$ Denominator factors: $7\cdot 2\cdot(x-2)\cdot(x-2)$

    LCD: $7\cdot 2\cdot (x-2)^2 = 14(x-2)^2$
- Try these on page 466: 5, 7, 13, 27, 29, 33, 37, 39 (Solutions start on page 306 of the solution manual)

7.4: Adding & Subtracting Rational Expressions with Unlike Denominators (p. 468)

- Find the LCD.
- Rewrite each rational expression (fraction) as an equivalent expression whose denominator is the LCD.
- Add or subtract numerators and write the sum or difference over the common denominator.
- Simplify if possible.
- Example: $\frac{9x}{x-10} - \frac{x}{x-3}$ LCD $= (x-10)(x-3)$

  We'll need to multiply each fraction in the numerator and denominator by the factor they need to get the LCD.

  $$\frac{9x}{x-10}\cdot\frac{(x-3)}{(x-3)} - \frac{x}{x-3}\cdot\frac{(x-10)}{(x-10)}$$

  $$\underbrace{\frac{9x(x-3)}{(x-10)(x-3)}} - \underbrace{\frac{x(x-10)}{(x-3)(x-10)}} \Rightarrow \frac{9x(x-3)-x(x-10)}{(x-10)(x-3)} \Rightarrow \frac{9x^2-27x-x^2+10x}{(x-10)(x-3)} \Rightarrow \frac{8x^2-17x}{(x-10)(x-3)} \Rightarrow \frac{x(8x-17)}{(x-10)(x-3)}$$

  Remember, order of multiplication doesn't matter.
- Try these on page 472: 7, 15, 21, 29, 37, 41, 45, 47, 51, 55 (Solutions start on page 310 of solution manual)

7.5: Solving Equations Containing Rational Expressions (p. 474)

- Multiply every term on both sides of the equation by the LCD to eliminate denominators.
- Remove any grouping symbols (distribute if necessary) and solve the resulting equation.
- Check the solution in the original equation (make sure your answer doesn't result in dividing by zero!)
- Example: $\frac{1}{x+3} = \frac{9}{x^2-9} - \frac{1}{x-3}$ $\Rightarrow$ LCD: $(x+3)(x-3)$

$$\frac{1}{x+3}\cdot(x+3)(x-3) = \frac{9}{(x+3)(x-3)}\cdot(x+3)(x-3) - \frac{1}{x-3}\cdot(x+3)(x-3)$$

$$\Rightarrow 1(x-3) = 9 - 1(x+3) \Rightarrow \underset{+x+3}{x-3} = \underset{+x+3}{9-x-3} \Rightarrow \frac{2x}{2} = \frac{15}{2} \Rightarrow x = \frac{15}{2}$$

- Try these on page 479: 3, 7, 15, 19, 31, 41, 47, 49 (Solutions start on page 319 of the solutions manual)

7.6: Proportion and Problem Solving with Rational Equations (p. 482)

- If you are given two proportions equal to each other, we can cross multiply.
  ie: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
- Example: $\frac{x+3}{2x+1} = \frac{6}{11} \Rightarrow (x+3)(11) = (6)(2x+1) \Rightarrow \underset{-11x\ -6}{11x+33} = \underset{-11x\ -6}{12x+6} \Rightarrow 27 = x$

Related Rate Word Problems

- Example, # 22 p. 492
  - Experienced: 3 hours
  - Apprentice: 6 hours
  - Together: ?
- Set up a table

	Hours to complete whole job	Part of job completed in one hour
Experienced	3 hours	$\frac{1}{3}$
Apprentice	6 hours	$\frac{1}{6}$
Together	x hours	$\frac{1}{x}$

In one hour: Experienced + Apprentice = Together

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{x}$$

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{x} \qquad LCD = 6x$$

$$\frac{1 \cdot 6x}{3} + \frac{1 \cdot 6x}{6} = \frac{1 \cdot 6x}{x}$$

$$\frac{\overset{2}{6x}}{3} + \frac{6x}{6} = \frac{6x}{x}$$

$$2x + x = 6 \Rightarrow \frac{3x}{3} = \frac{6}{3} \Rightarrow x = 2$$

It will take them 2 hours to construct the wall together.

- Example: #28 p. 492
  - set up a table using distance = rate × time

	distance	= rate	× time
flatland	300 miles	x	time
mountains	180 miles	x−20	time

time is the same (mentioned in problem)

flatland: $300 = x \cdot t \Rightarrow t = \frac{300}{x}$

mountains: $180 = (x-20) \cdot t \Rightarrow t = \frac{180}{x-20}$

- because the t's are equal, we can set the other parts of the equation equal to each other.

$$\frac{300}{x} = \frac{180}{x-20} \Rightarrow 300(x-20) = 180(x) \Rightarrow 300x - 6000 = 180x$$

cross multiply

$$\quad -300x \qquad -300x$$

$$\Rightarrow \frac{-6000}{-120} = \frac{-120x}{-120} \Rightarrow 50 = x$$

Rate in flatland $= x = 50 \frac{mi}{hr.}$

Rate in mountains $= x - 20 = 50 - 20 = 30 \frac{mi}{hr.}$

- Try these on page 491: 5, 7, 9, 11, 19, 23, 25, 29, 33, 45, 47, 51

(Solutions start on page 335 of the solutions manual.)

7.7: Simplifying Complex Fractions (p. 495)

Two methods to simplify:

① • Simplify numerator and denominator separately so
  – Write the numerator as one fraction and denominator as one fraction
• Perform the division of the two fractions (skip, flip, & multiply).
• Simplify if possible.
• Example: $\dfrac{\frac{1}{y}+\frac{3}{y^2}}{y+\frac{27}{y^2}}$

num: $\frac{1}{y}+\frac{3}{y^2}$ $(LCD = y^2) \Rightarrow \frac{1}{y}\left(\frac{y}{y}\right)+\frac{3}{y^2} \Rightarrow \frac{y}{y^2}+\frac{3}{y^2} = \frac{y+3}{y^2}$

denom: $y+\frac{27}{y^2}$ $(LCD = y^2) \Rightarrow \frac{y}{1}\left(\frac{y^2}{y^2}\right)+\frac{27}{y^2} \Rightarrow \frac{y^3}{y^2}+\frac{27}{y^2} = \frac{y^3+27}{y^2}$

NOTE: y^3+27 is a SUM OF CUBES!

$$a^3+b^3 = (a+b)(a^2+ab+b^2)$$

$$y^3+27 = \underset{a}{y^3}+\underset{b}{3^3} = (y+3)(y^2-3y+9)$$

Put everything back together...

$$\frac{\frac{y+3}{y^2}}{\frac{(y+3)(y^2-3y+9)}{y^2}} \Rightarrow \frac{y+3}{y^2}\cdot\frac{y^2}{(y+3)(y^2-3y+9)} \Rightarrow \frac{1}{y^2-3y+9}$$

skip, flip, & multiply!

② • Find the LCD of <u>all</u> of the fractions (in numerator <u>and</u> denominator, then multiply both the numerator and denominator by the LCD.
• Simplify.
• Example: $\dfrac{\frac{1}{y}+\frac{3}{y^2}}{y+\frac{27}{y^2}}$ LCD of <u>all</u> fractions: y^2

$$\left(\frac{\frac{1}{y}+\frac{3}{y^2}}{y+\frac{27}{y^2}}\right)\frac{y^2}{y^2} \Rightarrow \frac{\overbrace{\frac{1\cdot y\cdot y}{y}}^{y}+\overbrace{\frac{3\cdot y^2}{y^2}}^{3}}{\underbrace{y\cdot y^2}_{y^3}+\underbrace{\frac{27\cdot y^2}{y^2}}_{27}} \Rightarrow \frac{y+3}{\underbrace{y^3+27}_{\text{sum of cubes}}} \Rightarrow \frac{y+3}{(y+3)(y^2-3y+9)} \Rightarrow \frac{1}{y^2-3y+9}$$

sum of cubes: $a^3+b^3 = (a+b)(a^2-ab+b^2)$

Negative Exponents

- $x^{-1} = \frac{1}{x}$

  negative = put in denominator

- $\frac{1}{x^{-1}} = \frac{1}{\frac{1}{x}} = \frac{1}{1} \cdot \frac{x}{1} = x$

  (skip, flip, & multiply)

- $xy^{-1} = x \cdot \frac{1}{y} = \frac{x}{y}$
  - the exponent only applies to the variable or number it is directly behind. In this case, the (-1) exponent only applies to the y-variable.
- $(xy)^{-1} = \frac{1}{xy}$
  - Because the exponent is being applied to <u>everything</u> in the parenthesis, we will put everything in the parenthesis in the denominator.
- $2x(yz)^{-1} = \frac{2x}{yz}$

  (-1) exponent only for y & z

- Example: $\frac{5x^{-2} - 3y^{-1}}{x^{-1} + y^{-1}} \Rightarrow \underbrace{\frac{\frac{5}{x^2} - \frac{3}{y}}{\frac{1}{x} + \frac{1}{y}}}$

  now we can use one of the two methods to simplify.

  Method 1: num: $\frac{5}{x^2} - \frac{3}{y}$, LCD $= x^2y \Rightarrow \frac{5}{x^2}\left(\frac{y}{y}\right) - \frac{3}{y}\left(\frac{x^2}{x^2}\right) = \frac{5y - 3x^2}{x^2y}$

  denom: $\frac{1}{x} + \frac{1}{y}$, LCD $= xy \Rightarrow \frac{1}{x}\left(\frac{y}{y}\right) + \frac{1}{y}\left(\frac{x}{x}\right) \Rightarrow \frac{y + x}{xy}$

  together: $\frac{\frac{5y-3x^2}{x^2y}}{\frac{y+x}{xy}} \Rightarrow \frac{5y-3x^2}{x^2y} \cdot \frac{xy}{y+x} \Rightarrow \frac{5y-3x^2}{x(y+x)}$

  either way, same answer

  Method 2: LCD of EVERYTHING: x^2y

  $\left(\frac{\frac{5}{x^2} - \frac{3}{y}}{\frac{1}{x} + \frac{1}{y}}\right)\left(\frac{x^2y}{x^2y}\right) \Rightarrow \frac{\frac{5(x^2y)}{x^2} - \frac{3(x^2y)}{y}}{\frac{1(x \cdot x \cdot y)}{x} + \frac{1(x^2y)}{y}} \Rightarrow \frac{5y - 3x^2}{xy + x^2} \Rightarrow \frac{5y - 3x^2}{x(y+x)}$

  $x^2y = x \cdot x \cdot y$

- Try these on page 500: 1, 7, 11, 29, 31, 33, 41, 45, 49 (Solutions start on page 342 of the solutions manual).